Workshop/School on Stochastic PDEs, Mean Field Games and Biology

Gran Sasso Science Institute - L'Aquila (Italy) August, 8 – September, 9 2017

Abstracts of the talks

B. Perthame (UMPC – Paris, France)

T.B.A

A. Porretta (Univ. Rome Tor Vergata, Italy)

Mean Field Games: introduction and PDE methods

Mean field games theory describes Nash equilibria in the dynamic optimization of a large population of similar agents, where the individual strategy depends on the collective behavior through the distribution law of the states. Through the usual dynamic programming approach, this model leads to systems of PDEs where a backward Hamilton-Jacobi-Bellman equation is coupled with a forward Kolmogorov-Fokker-Planck equation. In this short course, I will give a brief description of the model and I will discuss the main features of the PDE systems such as existence, uniqueness and stability of solutions. Further issues will be discussed in other courses of the school.

Y. Achdou (Univ. Paris Diderot, France)

Numerical methods for mean field games

Recently, an important research activity on mean field games (MFGs for short) has been initiated by the pioneering works of Lasry and Lions: it aims at studying the asymptotic behavior of stochastic differential games (Nash equilibria) as the number n of agents tends to infinity.
At the limit, a given agent feels the presence of the others through the statistical distribution of the states. Assuming that the perturbations of a single agent's strategy does not influence the statistical states distribution, the latter acts as a parameter in the control problem to be solved by each agent. When the dynamics of the agents are independent stochastic processes, MFGs naturally lead to a coupled system of two partial differential equations (PDEs for short), a forward Fokker-Planck equation and a backward Hamilton-Jacobi-Bellman equation.

The latter system of PDEs has closed form solutions in very few cases only. Therefore, numerical simulation are crucial in order to address applications. The present mini-course will be devoted to numerical methods that can be used to approximate the systems of PDEs.

The mini-course will be organized as follows:

1) Monotone finite difference schemes;
2) Examples of applications;
3) Variational MFG and related algorithms for solving the discrete system of nonlinear equations.

B. Gess (Max Planck Leipzig, Germany)

Stochastic scalar conservation laws

In this lecture we will consider the recently developed theory of stochastic scalar conservation laws. A model example of this class of SPDE is given by the stochastic Burgers equation. In the first lecture we will recall aspects of the theory of deterministic scalar conservation laws. This will lead to the notions of entropy, kinetic and quasi-solutions. We will then derive appropriate notions of solutions for stochastic scalar conservation laws. In the second lecture, we will investigate the effect of well-posedness by noise in the setting of scalar conservation laws. We will start with (deterministic) conservation laws for which entropy solutions are non-unique and prove that certain random perturbations lead to uniqueness. In the third lecture, a regularizing effect of noise will be considered, that is, we will observe instances of stochastic scalar conservation laws for which solutions are more regular than in the non-perturbed, deterministic case.

M. Gubinelli (Univ. Bonn, Germany)

Weak universality and Singular SPDEs

Mesoscopic fluctuations of microscopic (discrete or continuous) dynamics can be described in terms of nonlinear stochastic partial differential equations which are universal: they depend on very few details of the microscopic model. Due to the extreme irregular nature of the random field sample paths, these equations turn out to not be well-posed in any classical analytic sense. I will review recent progress in the mathematical understanding of such singular equations and of their (weak) universality.

P. Jabin (Univ. of Maryland College Park, USA)

Oscillations in selection-mutation dynamics

We study the limit of many small mutations for a model of population dynamics where the population is structured by phenological traits and the various sub-populations compete for the same nutrients. In many such models, the main challenge in deriving the limit is to control possible time oscillations in the population. We explain how a control on such oscillations can be obtained directly from the stability of a corresponding dynamical system at a faster time scale.

B. Moll (Univ. Princeton, USA)

Mean Field Games in Economics

Mean Field Games are everywhere in economics. Why? Because heterogeneity is everywhere and therefore many interesting questions require thinking about distributions of economic variables. In particular, macroeconomists often use so-called "heterogeneous agent models" to understand the interactions between income and wealth distribution and aggregates like Gross Domestic Product (GDP). Classic examples are papers by Huggett (1993), Aiyagari (1994) and Krusell and Smith (1998). These models are really Mean Field Games: Individuals optimize taking as given the evolution of the wealth distribution (HJB equation), and the evolution of the wealth distribution is determined by individual saving behavior (Fokker-Planck equation). I will present a number of examples of such MFGs and discuss what is known about their properties. I will also list some open questions for future research with particular focus on the analysis and numerical solution of MFGs with common noise.

Background readings:
http://www.princeton.edu/~moll/PDE_macro.pdf
http://www.princeton.edu/~moll/HACT.pdf
http://www.princeton.edu/~moll/WIMM.pdf

T. Souganidis (Univ. of Chicago, USA)

T.B.A.

F. Santambrogio (Univ. Paris-Sud, France)

Variational Mean Field Games and Optimal Transport

In this mini-course, I will explain the analogies between the optimization problems lying behind some classes of variational mean-field games and the Monge-Kantorovich theory of optimal transport. In particular, the connection will be done with the kinetic energy minimization model introduced by Benamou and Brenier ; time-discretization will also be performed, reducing the problem to a sequence of optimization involving Wasserstein distances and recalling the so-called Jordan-Kinderleher-Otto scheme for evolution PDEs having an gradient flow structure.
These connections and ideas will then be used to obtain regularity results which are needed for a careful analysis of the models and for a rigorous passage from optimizers to equilibria. In particular, the first lecture will be devoted to an introduction to the main tools that we will use from optimal transport theory, the second will be devoted to methods originating from convex duality and the third to methods based on time-discretization.

P. Cardialiaguet (Univ. Paris Dauphine, France)

The master equation and its applications in mean field games.

The master equation is a kind of hyperbolic equation stated in the space of probability measures. It plays a central role in mean field game theory (MFG): it allows to formalize, for instance, the so-called MFG with common noise, in which the players are subject to a common uncertainty. It is also a key tool to understand the convergence to the MFG equilibrium of the system associated with a finite number of players as the number of players tends to infinity. We will discuss the structure of this equation (existence, uniqueness, characteristics), and its use in the mean field limit.

J. Calder (Univ. Minnesota, USA)

T.B.A.

N. Champagnat (INRIA, France)

T.B.A.