Semi-infinite quasi-Toeplitz matrices: analysis algorithms and applications

Dario A. Bini, Università di Pisa

Let $a(z) = \sum_{i=-\infty}^{+\infty} a_i z^i$ be a complex valued function defined for $z \in \mathbb{C}$, $|z| = 1$, such that $\sum_{i=-\infty}^{+\infty} |a_i| < \infty$. The semi-infinite matrix $T(a) = (t_{i,j})_{i,j \in \mathbb{Z}^+}$ is said *Toeplitz matrix* associated with $a(z)$ if $t_{i,j} = a_{j-i}$. Typically, Toeplitz matrices are encountered when some shift invariance property is satisfied in a mathematical model. In particular, many queuing models are described by *quasi-Toeplitz* (QT) matrices, that is, matrices of the form $A = T(a)+E$ where E is a compact correction. For instance, in the random walk along a half-line, the probability transition matrix is the sum of a semi-infinite tridiagonal Toeplitz matrix and a correction E which is nonzero only in the entry (1,1). More complex situations are encountered if the domain is the quarter plane like in the Tandem Jackson Queue. Here, the probability matrix is block Toeplitz with Toeplitz blocks plus finite rank corrections.

In this framework, the main computational problems are: computing matrix functions, solving polynomial matrix equations, and solving linear systems where the input matrices are QT.

In this talk, after pointing out the role of QT matrices in certain applications, we introduce some matrix norms, which make the class of QT matrices a Banach algebra and at the same time, are computationally tractable. Then we introduce the class of QT matrices representable by a finite number of parameters together with a matrix arithmetic on this class. This way, we may approximate QT matrices by a finite number of parameters in the same way as real numbers are approximated by floating point numbers.

Finally, we introduce algorithms for the solution of the main computational problem encountered in this framework like computing the inverse matrix by means of the Wiener-Hopf factorization, computing the matrix exponential, solving quadratic matrix equations encountered in Quasi–Birth-Death stochastic processes where matrix coefficients are QT matrices.