

New trends in control of evolution systems

LIST OF ABSTRACTS

Davide Barilari

Title: CURVATURE TERMS IN THE SMALL TIME HEAT KERNEL EXPANSION FOR A CLASS OF HYPOELLIPTIC HÖRMANDER OPERATORS

Abstract: We consider the heat equation associated with some classes of second order hypoelliptic Hörmander operators. We discuss the small time heat kernel expansion on the diagonal, giving in particular a geometric characterization of the coefficients in terms of the divergence of the drift field and the curvature-like invariants of the optimal control problem associated with the diffusion operator.

Francesca Ceragioli

Title: DISCONTINUITIES IN OPINION DYNAMICS

Abstract: In the latest years the problem of reaching consensus has been deeply studied in the context of distributed control systems over networks. When such studies are applied to social dynamics and to opinion dynamics in particular, the counterpart of consensus, namely disagreement, has an identical interest. In this talk I will present and analyse some models in which discontinuities naturally enter, giving rise to clusterization instead of consensus.

Wei Cheng

Title: GLOBAL GENERALIZED CHARACTERISTICS FOR THE DIRICHLET PROBLEM FOR HAMILTON-JACOBI EQUATIONS

Abstract: We study the nonhomogeneous Dirichlet problem for first order Hamilton-Jacobi equations associated with Tonelli Hamiltonians on a bounded domain Ω of \mathbb{R}^n assuming the energy level to be supercritical. First, we show that the viscosity (weak KAM) solution of such a problem is Lipschitz continuous and locally semiconcave in Ω . Then, we analyze the singular set of a solution showing that singularities propagate along suitable curves, the so-called generalized characteristics, and that such curves stay singular unless they reach the boundary of Ω . Moreover, we prove that the latter is never the case for mechanical systems and that singular generalized characteristics converge to a critical point of the solution in finite or infinite time. Finally, under stronger assumptions for the domain and Dirichlet data, we are able to conclude that solutions are globally semiconcave and semiconvex near the boundary. This is a joint work with Piremarco Cannarsa, Marco Mazzola and Kaizhi Wang.

Giuseppe Floridia

Title: CARLEMAN ESTIMATES AND APPLICATIONS TO INVERSE PROBLEMS FOR TRANSPORT EQUATIONS

Abstract: In this talk we introduce a Carleman estimate for a transport equation, that is obtained using a piecewise continuous weight function.

This kind of estimate permits to obtain an observability inequality useful for the applications to inverse problems.

This is a joint work with Piermarco Cannarsa (University of Rome “Tor Vergata”) and Masahiro Yamamoto (The University of Tokyo).

Genni Fragnelli

Title: CARLEMAN ESTIMATES AND NULL CONTROLLABILITY FOR A DEGENERATE POPULATION MODEL

Abstract: We deal with a degenerate model describing the dynamics of a population depending on time, on age and on space. We assume that the degeneracy can occur at the boundary or in the interior of the space domain and we focus on null controllability results. To this aim, we prove first Carleman estimates for the associated adjoint problem, then we prove the existence of a null control function localized in the interior of the space domain

Paola Loreti

Title: WAVE-WAVE WITH MEMORY

Abstract: The talk will focus on the presence of an integro-differential term (fading memory) in one of the equations of a weakly coupled system of wave-wave type.

We provide observability estimates for the solution of the adjoint system, and then reachability results for sufficiently large time.

Joint work with Daniela Sforza.

Pierdomenico Pepe

Title: STABILIZATION OF RETARDED SYSTEMS: A LYAPUNOV-BASED SAMPLED-DATA APPROACH

Abstract: Lyapunov-based control methodologies are shown for the class of systems described by retarded functional differential equations. Discontinuous state feedbacks are investigated, with a sampled-data approach. Both local and global stabilizers are presented, for both exact and practical convergence. Implementation issues are studied, and a spline approximation scheme is proposed for the approximation of infinite dimensional state feedbacks, which makes use of a finite set of sampled data. Examples are provided.

Cristina Pignotti

Title: STABILITY RESULTS FOR TIME DELAYED EVOLUTION EQUATIONS

Abstract: We consider abstract evolution equations with a delay feedback and assume that the C_0 -semigroup associated with the undelayed part of the model is exponentially stable. Then, we show that the whole system retains this property when an appropriate condition on the time delay feedback coefficient $k(t)$ is satisfied. Some concrete applications of the abstract results are also illustrated.

Daniela Sforza

Title: HIDDEN REGULARITY FOR EVOLUTION EQUATIONS WITH MEMORY

Abstract: The purpose of the talk is to show hidden regularity results for linear and nonlinear wave equations with integral terms of convolution type. Under general assumptions on the integral kernel we define the trace of the normal derivative of the solution showing a regularity result. In such a way we extend to integrodifferential equations well-known results available in the literature for wave equations without integral terms.

Joint works with Paola Loreti.

Bucci Francesca

Title: FROM THE MOORE-GIBSON-THOMPSON EQUATION TO WAVE EQUATIONS
WITH MEMORY AND BACK
AGAIN: A REGULARITY ANALYSIS

Abstract: In this talk we will deal with the linearization of a Partial Differential Equation (PDE) model for ultrasound waves, a third-order in time PDE referred to in the literature as the Moore-Gibson-Thompson (MGT) equation. Unlike the case of classical models in nonlinear acoustics such as the Westervelt and the Kuznetsov equations, whose linearized dynamics -- that is the strongly damped wave equation -- is described by an analytic semigroup, the MGT equation has a hyperbolic character. Thus, the regularity of solutions to initial/boundary value problems corresponding to boundary data which are, e.g., square integrable in time and space, as well as the sharp regularity of boundary traces, are relevant topics of investigation.

In the talk we will present several results established for the MGT equation when supplemented with Dirichlet or Neumann boundary conditions, by embedding it in a family of wave equations with memory, and affine term depending on a parameter. The regularity analysis pertaining to these integro-differential equations is of independent interest; furthermore, the unified method of proof utilized -- operator-theoretic for the most part, yet depending crucially on the literature on (linear, memoryless) wave equations -- allows for the derivation of boundary regularity results for both models.

(The talk is based on ongoing joint work with Luciano Pandolfi (Politecnico di Torino, Italy).)

Cristina Urbani

Joint work with F. Alabau, P. Cannarsa

Title: LOCAL CONTROLLABILITY FOR LINEAR EVOLUTION EQUATIONS WITH BILINEAR CONTROL

Abstract: In control theory there are few results of controllability of equations via bilinear control with respect to boundary or localized controls. The reason is that, contrarily to boundary and linearized control problems, the presence of a control that multiplies the state of the equation brings non linearity in the system. Therefore, there are no direct methods to approach these problems and usually the strategy consists in linearizing the equation around a given equilibrium or trajectory, studying the controllability of this simplified problem and then, if it is possible, deducing the desired property for the non linear system through an inverse mapping theorem.

I will present a result of bilinear controllability of evolution equations with an operator generating an analytical semigroup. Finally, I will apply the abstract result to some examples.

Silvia Romanelli

Title: DEGENERATE ELLIPTIC OPERATORS AND REGULARITY OF THEIR ASSOCIATED SEMIGROUPS

Evolution equations associated with degenerate elliptic operators naturally arise in many problems coming from the applied sciences. Here we consider the case of an operator A in divergence, or nondivergence, form defined in a bounded domain Ω of R^n with smooth boundary $\partial\Omega$. For simplicity we deal with $A = \nabla \cdot (a\nabla)$ or $A = a\Delta$. In our setting, degeneracy means that a vanishes on (a part of) $\partial\Omega$, or on an interior point of Ω . The main aim will be to show that, under additional assumptions, the closure of A with a suitable domain generates an analytic semigroup in different types of spaces.

Masahiro Yamamoto

Titolo: OBSERVABILITY INEQUALITIES AND COEFFICIENT INVERSE PROBLEMS FOR ANISOTROPIC HYPERBOLIC EQUATION WITH MEMORY TERM

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with smooth boundary $\partial\Omega$. For $T > 0$, we consider the following integro-differential hyperbolic equations:

$$\partial_t^2 u(x, t) = \sum_{i,j=1}^n \partial_i(a_{ij}(x)\partial_j u) + \int_0^t \sum_{|\alpha| \leq 2} b_\alpha(x, t, \tau) \partial_x^\alpha u(x, \tau) d\tau, \quad x \in \Omega, 0 < t < T$$

with

$$u|_{\partial\Omega} = 0, \quad 0 < t < T.$$

Here and henceforth let $\alpha = (\alpha_1, \dots, \alpha_n) \in (\mathbb{N} \cup \{0\})^n$ be a multi-index and we set $|\alpha| = \alpha_1 + \dots + \alpha_n$, $\partial_i = \frac{\partial}{\partial x_i}$, $1 \leq i \leq n$, $\partial_t = \frac{\partial}{\partial t}$, $\partial_x^\alpha = \partial_1^{\alpha_1} \dots \partial_n^{\alpha_n}$.

Moreover let Γ be a suitable subboundary of $\partial\Omega$.

We discuss

- **Observability inequalities**

Estimate the energy by extra data ∇u on $\Gamma \times (0, T)$.

- **Coefficient inverse problems.**

For suitably given initial values, determine all or some of $a_{ij}(x)$ by extra data ∇u on $\Gamma \times (0, T)$.

Although there are rich references in the isotropic case $a_{ij} = 0$ if $i \neq j$, the anisotropic principal part causes difficulties. We overcome them by a new Carleman estimate.

The work is based on the joint work with Professor Paola Loreti and Professor Daniela Sforza (Sapienza Università di Roma).